

OPTIMUM SELECTION INDICES FOR CONTINUOUS SELECTION PROGRAMMES

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SUMMARY

Selection indices including several sources of information do not maximize genetic response after the first generation of selection. Generation of linkage disequilibrium induced by selection reduces both genetic and phenotypic variances. In this paper a procedure to obtain selection index weights to maximize response in the limit, by accommodating the changes in genetic and phenotypic variances induced by selection is described. Benefits in response were very small for the example considered.

INTRODUCTION

The use of the performance on relatives to maximize response to selection can be accomplished by the construction of selection indices (Lush, 1947). If several sources of information are used in a linear weighted combination of observed measurements, then selection indices maximize response only in the first generation of selection. An immediate consequence of selection is that genetic and phenotypic variances are reduced. For populations of infinite size and under the assumptions of the infinitesimal gene effect model, the changes in genetic variance are entirely due to the generation of linkage disequilibrium. Bulmer (1971) has shown that after a few cycles of selection a steady-state equilibrium is reached where the new disequilibrium caused by selection is balanced by free recombination. At this point there are no further changes in genetic or phenotypic variances if accuracy and selection intensity are kept constant. Wray and Hill (1989) and Gomez-Raya and Burnside (1990) have discussed the changes in genetic variance and associated reduction in response under index selection including information on relatives. The objective of this paper is to derive selection index weights to maximize response for the steady-state equilibrium situation.

EXAMPLE : SELECTION BASED ON INDIVIDUAL AND ONE PARENT

Consider the simple case of selection based on an individual and one of its parents. The selection index for this example is :

$$I = b_1 P_1 + b_2 P_2$$

where b_1 : weight for individual performance
 P_1 : individual performance
 b_2 : weight for parent performance
 P_2 : parent performance

Let $x = b_2/b_1$. Then the selection index becomes

$$I = P_1 + x P_2$$

Accuracy of evaluation using selection index theory is

$$r = (1 + .5x)/(M + x)^{.5}$$

where $M = (1 + x^2)/h^2$ and h^2 = heritability

From Gomez-Raya and Burnside (1990) the genetic variance in the limit is

$$\sigma_{A_L}^2 = \sigma_{A_0}^2 / (1 + k r_L^2)$$

where $k = i (1-x^*)$

i = selection intensity

x^* = normal deviate at truncation point

$\sigma_{A_0}^2$ = genetic variance in the base population

and r_L = accuracy of selection in the limit.

The heritability in the limit can be obtained as

$$h_L^2 = (-b + (b^2 - 4ac)^{.5}) / 2a$$

where $a = x + [(1 - h_0^2) k (1 + .25 x^2 + x)]$

$b = 1 + x^2 - h_0^2 x$

and $c = -h_0^2 (1 + x^2)$.

The response in the limit is then

$$R_L = i \sigma_{A_L} r_L = i \sigma_{A_0} / [(1/r_L^2) + k]^{.5}$$

OPTIMUM SELECTION INDICES FOR THE STEADY-STATE EQUILIBRIUM

To obtain the selection index weights for the steady-state equilibrium we differentiate R_L with respect to x . In this way changes in accuracy, heritability and genetic variance induced by the action of selection are accounted for in the derivation of the index weights. Equating the resulting expression to 0, the following equation is obtained

$$h_L^4 (1 - .5x) - h_L^2 (1 - 2x) - [(1 + .5x)(1 + x^2)(d(h_L^2)/dx)] = 0$$

where the derivation of $(d(h_L^2)/dx)$ is shown in the appendix.

To solve this equation for x the bisection method was used because further differentiation is not required.

A comparison of responses in the limit from the traditional index (x_T) with that from steady-state equilibrium index (x_S) was achieved by computing the percentage increase in response of the steady-state index over the traditional index. That is,

$$PR_L = [(R_S/R_T) - 1] \times 100$$

where R_T = is the response in the limit of the traditional index

R_S = is the response in the limit of the steady-state index.

Table 1. Traditional selection index weights (x_t) and steady-state equilibrium index weights (x_s) and percentage increase in response at the limit (PR_L) for different percentage selected (p) and heritability in the base population (h_0^2).

h_0^2	x_t	p=1%		p=20%		p=50%	
		x_s	PR_L	x_s	PR_L	x_s	PR_L
.10	.4615	.4648	.0003	.4644	.0002	.4639	.0001
.20	.4211	.4317	.0027	.4305	.0022	.4291	.0016
.30	.3784	.3985	.0088	.3964	.0074	.3938	.0056
.40	.3333	.3638	.0191	.3607	.0162	.3568	.0125
.50	.2857	.3263	.0325	.3223	.0277	.3171	.0216
.60	.2353	.2844	.0463	.2795	.0395	.2732	.0309
.70	.1818	.2360	.0554	.2304	.0471	.2233	.0368
.80	.1250	.1777	.0521	.1720	.0439	.1648	.0340
.90	.0645	.1035	.0287	.0989	.0238	.0932	.0180

Steady-state indices gave more relative importance to the parent performance than traditional indices (table 1). The percentage of increase in the response at the limit (PR_L) was very small ranging from .0001% to .0554%. This result is consistent with Sales and Hill (1976) that selection indices are robust to incorrect estimates of genetics parameters. The increase in accuracy and response by the use of parent's performance is small in both the first and in the limit generations. Optimum selection indices accommodating the changes in genetic and phenotypic variances following repeated cycles of selection did not significantly increase response in the limit with respect to that attained by traditional index procedures (x_t). Benefits of optimum selection indices in the steady-state equilibrium have been shown to be negligible for the example considered. If a larger number of sources of information, such as several traits, are included in the index, the advantages of optimum selection indices at the limit could be larger. Obtaining optimum selection index weights becomes algebraically more and more complex as the number of sources of information increases in the index. In practical situations, BLUP (Best Linear Unbiased Predictor) is preferred to the selection index method to evaluate individuals. Mixed model methods appear to be unbiased by selection of animals to be parents under repeated cycles of selection in simulation work (Sorensen and Kennedy, 1984) provided the model is correct. However, multiple trait BLUP evaluations give breeding values for each trait included in the analysis (Schaeffer, 1984). Combining the information from different traits in an optimum steady-state selection index would result in larger selection responses in continuous selection programmes.

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APPENDIX

Derivative of heritability at the limit with respect to x .

$$d(h_L^2)/dx = \frac{2a [A + (B/C)] - [D E]}{4a^2}$$

where A = d(-b)/dx

B = (d(b²)/dx) - (d(4ac)/dx)

C = 2 (b² - 4ac)^{.5}

D = d(2a)/dx

E = -b +(b² -4ac)^{.5}

(d(a)/dx) = 1 + [(1-h₀²) k (1 +.5x)]

(d(b²)/dx) = 2 (2x- h₀²)(1 +x² -h₀²x)

(d(c)/dx) = -2xh₀²

(d(2a)/dx) = 2(d(a)/dx)

(d(4ac)/dx) = 4 [a (d(c)/dx) + c (d(a)/dx)]